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Self-consistent theory of spin–phonon interactions in ferromagnetic semiconductors

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Abstract. A Green function technique is used to study the effects of spin–phonon interactions in ferromagnetic semiconductors. The spin wave energy and the transverse damping for two different spin–phonon interaction mechanisms are evaluated for the first time beyond the RPA. The results are valid below and above T_c. The temperature dependence of these quantities is discussed, and is found to be in agreement with the experimental data.

1. Introduction

In recent years, elementary excitations of the lattice and spin systems of solids, i.e. phonons and magnons, respectively, have been the subject of extensive investigations by means of light scattering (Güntherodt and Zeyher [1], Wakamura and Arai [2]). A new 'degree of freedom' is added to the usual phonon Raman scattering (RS) in solids by investigating magnetically ordered materials, in particular, ferromagnetic semiconductors (FMS) which exhibit interactions between the phonon and spin systems. Investigations in this direction have been addressed to magnetic and semiconducting Cd–Cr spinels, the europium chalcogenides and magnetic insulators such as VI_2.

The spin–phonon interaction in FMS has not, theoretically, been so intensively studied. Suzuki and Kamimura [3] developed within the molecular-field approximation (MFA) a theory of phonon RS in magnetic crystals, and Suzuki [4] applied it to phonon RS in europium sulphide. Inelastic light scattering from non-zone-centre phonons in the magnetic phases of the europium chalcogenides is interpreted by Safran *et al* [5] in terms of two-spin correlation functions. Ousaka *et al* [6, 7] studied the spin-assisted phonon RS in EuTe and concluded that the 5d spin–orbital interaction modulated by the lattice displacement is dominant in the RS process of EuTe. However, the experimental results of Güntherodt and Zeyher [1] do not confirm the conclusions of these model calculations. Wesselinowa [8] discussed the influence of the spin–phonon interaction on the spin polarizability, i.e. on the longitudinal damping $\gamma^{zz}(\mathbf{k})$ and on the dynamic structure factor $S^{zz}(\mathbf{k}, E)$, for FMS using a Green function technique for the first time beyond the random phase approximation (RPA). The theoretical results were applied to $CdCr_2Se_4$ [8] and to EuS [9]. The effects of the magnetic ordering on the phonon spectrum and on the phonon damping in FMS were studied in [10].

The aim of the present paper is to observe the spin wave energies of FMS beyond the RPA including the spin–phonon interaction.

2. Model and method

It is known that ferromagnetic semiconductors are well described by the so-called s–f model [11]. The Hamiltonian of the system may be written as

$$H = H_{\mathrm{m}} + H_{\mathrm{e}} + H_{\mathrm{me}} + H_{\mathrm{p}} + H_{\mathrm{mp}}. \tag{1}$$

H_{m} is the Heisenberg Hamiltonian for the ferromagnetically ordered f electrons:

$$\begin{aligned} H_{\mathrm{m}} &= -\frac{1}{2}\sum_{i,j} J_{ij} S_i S_j - g\mu_{\mathrm{B}} H \sum_i S_i^z \\ &= -\frac{1}{2}\sum_q J_q (S_q^z S_{-q}^z + S_q^- S_q^+) - g\mu_{\mathrm{B}} H \sqrt{N} S_0^z. \end{aligned} \tag{2}$$

H_{e} represents the usual Hamiltonian of the conduction band electrons:

$$H_{\mathrm{e}} = \sum_{q,\sigma} (\epsilon_{q\sigma} - \mu) c_{q\sigma}^+ c_{q\sigma} + \frac{1}{2} \sum_{\substack{q,k',k'' \\ \sigma,\sigma'}} v(q) c_{k''-q\sigma}^+ c_{k'+q\sigma'}^+ c_{k'\sigma'} c_{k''\sigma} \tag{3}$$

where $v(q)$ is the Coulomb interaction, $\sigma = \pm 1$.

The operator H_{me} couples the two subsystems (2) and (3) by an intra-atomic exchange interaction:

$$H_{\mathrm{me}} = -\frac{I}{2N} \sum_{q,p} [S_{q-p}^+ c_{p-}^+ c_{q+} + S_{q-p}^- c_{p+}^+ c_{q-} + S_{q-p}^z (c_{p+}^+ c_{q+} - c_{p-}^+ c_{q-})] \tag{4}$$

where I is the constant interaction energy.

H_{p} represents the usual Hamiltonian of the lattice vibrations:

$$H_{\mathrm{p}} = \frac{1}{2} \sum_q (P_q P_{-q} + \omega_q^2 Q_q Q_{-q}) \tag{5}$$

where Q_q, P_q and ω_q are the normal coordinate, momentum and frequency of the lattice mode with wave-vector q, respectively. The vibrational normal coordinate Q_q and the momentum P_q can be expressed in terms of phonon creation and annihilation operators:

$$\begin{aligned} Q_q &= (2\omega_q)^{-1/2} (a_q + a_{-q}^+) \\ P_q &= \mathrm{i}(\omega_q/2)^{1/2} (a_q^+ - a_{-q}). \end{aligned}$$

H_{mp} describes the interaction of the spins with the phonons:

$$\begin{aligned} H_{\mathrm{mp}} &= -\frac{1}{2} \sum_{q,p} \bar{F}(p,q) Q_{p-q} (S_q^z S_{-p}^z + S_q^- S_p^+) \\ &= -\frac{1}{2} \sum_{q,p} F(p,q) (a_{p-q} + a_{q-p}^+) (S_q^z S_{-p}^z + S_q^- S_p^+). \end{aligned} \tag{6}$$

The acoustic phonons are coupled to the ferromagnetic system via the constant

$$\bar{F}(p,q) = \frac{1}{N^{1/2}} \sum_h \frac{1}{h}(e_{p-q} \cdot h) J'(h)(\exp(\mathrm{i}p \cdot h) + \exp(\mathrm{i}q \cdot h))$$

$$F(p,q) = \bar{F}(p,q)/(2\omega_{p-q})^{1/2}.$$

The summation extends over the vector $r_i - r_j = h$ connecting all possible pairs of spin sites in the crystal, and e_p is the polarization of the phonon with wavenumber p.

The retarded Green function to be calculated is defined in matrix form as

$$\tilde{G}_k(t) = -\mathrm{i}\theta(t)\langle[B_k(t), B_k^+]\rangle. \tag{7}$$

The operator B_k stands symbolically for the set $S_k^+, \sum_p c_{p+k+}^+ c_{p-}$. For the approximate calculation of the Green function (7) we use a method proposed by Tserkovnikov [12], which is appropriate for spin problems. After a formal integration of the equation of motion for the Green function one obtains

$$\tilde{G}_k(t) = -\mathrm{i}\theta(t)\langle[B_k, B_k^+]\rangle \exp(-\mathrm{i}E_k(t)t)$$

where

$$E_k(t) = \epsilon_k - \frac{\mathrm{i}}{t}\int_0^t \mathrm{d}t' t' \left(\frac{\langle[j_k(t), j_k^+(t')]\rangle}{\langle[B_k(t), B_k^+(t')]\rangle} - \frac{\langle[j_k(t), B_k^+(t')]\rangle\langle[B_k(t), j_k^+(t')]\rangle}{\langle[B_k(t), B_k^+(t')]\rangle^2} \right) \tag{8}$$

with the notation $j_k = [B_k, H_{\mathrm{int}}]$. The time-independent term

$$\epsilon_k = \langle[[B_k, H], B_k^+]\rangle / \langle[B_k, B_k^+]\rangle \tag{9}$$

gives the spin wave energy in the generalized Hartree–Fock approximation. The time-dependent term includes the damping effects.

3. The spin wave energy

We obtain from (9) for the spin wave energy in the generalized Hartree–Fock approximation

$$E_{1/2}(k) = 0.5\left[\epsilon_{11} + \epsilon_{22} \pm \sqrt{(\epsilon_{11} - \epsilon_{22})^2 + 4\epsilon_{12}\epsilon_{21}}\right] \tag{10}$$

with

$$\epsilon_{11} = g\mu_B H + \frac{1}{2\langle S^z\rangle}\left\{\frac{1}{N}\sum_q (J_q - J_{k-q})(2\langle S_q^z S_{-q}^z\rangle + \langle S_q^- S_q^+\rangle)\right.$$

$$+ \frac{I}{N^2}\sum_{q,p}(\langle S_{p-q}^- c_{p+}^+ c_{q-}\rangle + \langle S_{q-p}^z c_{p+}^+ c_{q+}\rangle - \langle S_{q-p}^z c_{p-}^+ c_{q-}\rangle)$$

$$+ \frac{1}{N}\sum_q (F_q^2/\omega_0)(\langle S_q^z S_{-q}^z\rangle + \langle S_q^- S_q^+\rangle)(2\langle S_q^z S_{-q}^z\rangle$$

$$+ \langle S_q^- S_q^+ \rangle - 2\langle S_{k-q}^z S_{q-k}^z \rangle - \langle S_{k-q}^- S_{k-q}^+ \rangle + \langle S^z \rangle) \Bigg\}$$

$$\epsilon_{12} = \frac{I}{2\rho N^2} \Bigg\{ \sum_{q,p} (\langle S_{p-q}^z c_{q-}^+ c_{p-} \rangle - \langle S_{q-p-k}^z c_{p+k+}^+ c_{q+} \rangle) - \frac{1}{2} \sum_{q,p} (\langle S_{p-q}^- c_{q+}^+ c_{p-} \rangle + \langle S_{k-q+p}^- c_{p+k+}^+ c_{q-} \rangle) \Bigg\}$$

$$\epsilon_{21} = \frac{I}{2\langle S^z \rangle N^2} \Bigg\{ \sum_{q,p} (\langle S_{k+p-q}^z c_{p+k-}^+ c_{q-} \rangle - \langle S_{p-q}^z c_{p+}^+ c_{q+} \rangle) - \frac{1}{2} \sum_{q,p} (\langle S_{k+p-q}^+ c_{p+k-}^+ c_{q+} \rangle + \langle S_{q-p}^+ c_{p-}^+ c_{q+} \rangle) \Bigg\}$$

$$\epsilon_{22} = 2\mu_B H + \frac{I}{2\rho N^2} \sum_{q,p} (\langle S_{q-p}^+ c_{p-}^+ c_{q+} \rangle + \langle S_{q-p}^z c_{p+}^+ c_{q+} \rangle - \langle S_{q-p}^z c_{p-}^+ c_{q-} \rangle) \tag{11}$$

where

$$\langle S_q^- S_q^+ \rangle = (\langle S^z \rangle /2)[(\epsilon_{11}(q)/E_q)\coth(E_q/2k_B T) - 1].$$

The matrix elements $\epsilon_{ij}(k)$ of the spin wave energy below T_c are in the RPA, where we have neglected the transverse correlation functions $\langle S_q^- S_q^+ \rangle$ and decoupled the longitudinal correlation functions $\langle S_q^z S_{-q}^z \rangle \to \langle S^z \rangle^2 \delta_{q0}$,

$$\begin{aligned} \epsilon_{11} &= g\mu_B H + \langle S^z \rangle (J_{\text{eff}} - J_k) + I\rho \\ \epsilon_{12} &= -I\langle S^z \rangle \\ \epsilon_{21} &= -I\rho \\ \epsilon_{22} &= 2\mu_B H + I\langle S^z \rangle \end{aligned} \tag{12}$$

where

$$J_{\text{eff}} = J_0 + \Delta J_{\text{sp}}$$
$$\Delta J_{\text{sp}} = \lim_{q \to 0} (F_q^2 \langle S^z \rangle / 2\omega_q).$$

Therefore the spin–phonon interaction causes below T_c a renormalization of the spin–spin interaction constant $J_0 \to J_{\text{eff}}$, which is now temperature dependent. Above T_c ΔJ_{sp} is zero.

ρ is the conduction-electron magnetization and is given by

$$\rho = \frac{n_+ - n_-}{2N} = \frac{1}{2N} \sum_{q,\sigma} \sigma \langle c_{q\sigma}^+ c_{q\sigma} \rangle \qquad \sigma = \pm 1 \tag{13}$$

where n_+ and n_- are the numbers of conduction electrons in the spin-up and spin-down bands, respectively. For the calculation of ρ we must define a one-electron Green function by $G_\sigma(k) = \langle\langle c_{k\sigma}; c_{k\sigma}^+ \rangle\rangle$. The electron energy is obtained as

$$\epsilon_\sigma(k) = \epsilon_k - \mu - \sigma(\mu_B H + 0.5I\langle S^z \rangle) + \sum_{k',\sigma'} [v(0) - v(k-k')\delta_{\sigma\sigma'}] \langle c_{k'\sigma'}^+ c_{k'\sigma'} \rangle \tag{14}$$

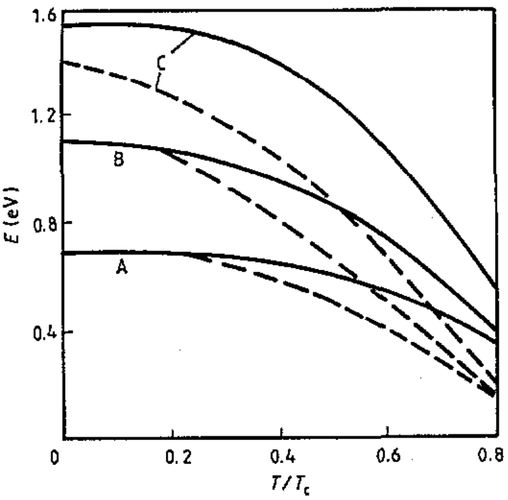

Figure 1. Temperature dependence of the spin wave energy E_1 for EuO (——) and E_2 for EuS (– – –) for different spin–phonon interaction constants: A, $F = 0$, B, 0.2 eV; C, 0.5 eV.

where ϵ_k is the conduction band energy in the paramagnetic state and μ is the chemical potential. For a simple cubic lattice and next-neighbour interaction ϵ_k is equal to

$$\epsilon_k = -\frac{W}{3}(\cos k_x a + \cos k_y a + \cos k_z a)$$

where W is the conduction band width. For the correlation function we have

$$\langle c^+_{q\sigma} c_{q\sigma} \rangle = 1/[\exp(\epsilon_{q\sigma}/k_B T) + 1].$$

The electron polarization ρ is maximum for $W = 0$ [13].

For the localized-spin magnetization $\langle S^z \rangle$ we obtain

$$\langle S^z \rangle = \frac{1}{N}\sum_k \{(S + \tfrac{1}{2})\coth[(S + \tfrac{1}{2})\beta E_k] - \tfrac{1}{2}\coth(\tfrac{1}{2}\beta E_k)\} \qquad (15)$$

where $E_k = E_1(\boldsymbol{k})$, $\beta = 1/k_B T$. $\langle S^z \rangle$ is maximum for $W = 0$. With increasing W, $\langle S^z \rangle$ and T_c decrease. If W is constant and I increases, then T_c increases too [13].

The spin wave energy $E_k = E_1(\boldsymbol{k})$ was calculated numerically taking parameters for EuO [11, 14, 15] (I = 0.2 eV, $S = 7/2$, W = 2 eV, T_c = 69.5 K, J/k_B = 0.55 K, J'/k_B = 0.15 K, $H = 0$, $\boldsymbol{k} = 0.2\pi$, 0.2π, 0.2π) and for EuS (I = 0.2 eV, $S = 7/2$, W = 0.9 eV, T_c = 16.5 K, J/k_B = 0.22 K, $J'/k_B = -0.10$ K, $H = 0$, $\boldsymbol{k} = 0.2\pi$, 0.2π, 0.2π) for different temperature T, spin–phonon interaction constant $F \equiv F(\boldsymbol{q})$ and Coulomb interaction U ($U \equiv 0.5v(\boldsymbol{q})$) values. J and J' are the spin–spin interaction constants between nearest and next-nearest neighbours, respectively. For the numerical calculations the following approximations are used: $\langle S^z_q S^z_{-q} \rangle = \langle S^z \rangle^2 \delta_{q0}$, $\langle S^z_{q-p} c^+_{p-} c_{q-} \rangle = \langle S^z \rangle \langle c^+_{q-} c_{q-} \rangle \delta_{pq}$, $\langle S^+_{q-p} c^+_{p-} c_{q+} \rangle = \langle S^+ \rangle \langle c^+_{q-} c_{q+} \rangle = 0$ and $\langle S^-_{p-q} c^+_{q+} c_{p-} \rangle = \langle S^- \rangle \langle c^+_{q+} c_{q-} \rangle = 0$. The results for U = 0.1 eV are demonstrated in figures 1 and 2. The spin wave energy $E(\boldsymbol{k})$ increases with U and F.

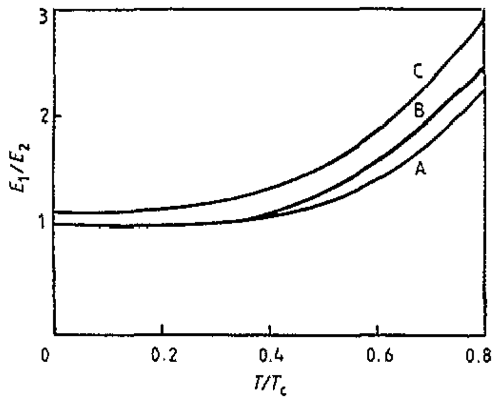

Figure 2. Temperature dependence of the ratio E_1/E_2 (E_1 for EuO and E_2 for EuS) for different spin–phonon interaction constants: A, $F = 0$; B, 0.2 eV; C, 0.5 eV.

At low temperatures the difference between the spin wave energies of EuO and EuS is very small. It grows with increasing T and increasing spin–phonon interaction F. So we have found that for $T \leqslant T_c$ the spin wave energies in EuS are small compared with EuO: for $F = 0.5$ eV and $T = 0.8T_c$ E_k in EuS is smaller by about a factor of three compared with EuO. Above T_c the spin wave energies decrease very slowly—they are nearly temperature independent. The results are in very good agreement with the experimental data of EuO and EuS [14, 16]. It is concluded that the spin–phonon interaction must be considered in order to obtain correct results in FMS in accordance with [9].

4. The damping

In order to obtain the spin wave damping caused by the spin–phonon interaction we consider approximately the integral term in (8). In our calculations we use the approximate dynamics $S_k(t) \simeq S_k \exp(-iE_k t)$, $c_{k\sigma}(t) \simeq c_{k\sigma} \exp(-i\epsilon_{k\sigma} t)$ and $a_k(t) \simeq a_k \exp(-i\omega_k t)$ where E_k and $\epsilon_{k\sigma}$ are from (10) and (14), respectively, and $\omega_k = vk$. This assumption takes the generalized Hartree–Fock approximation as a starting approximation.

Calculations yield the following expression for the transverse damping γ_k:

$$\gamma(k) = \gamma_{ss}(k) + \gamma_{sf}(k) + \gamma_{sp}(k). \tag{16}$$

γ_{ss} is the damping part which comes from the spin–spin interaction

$$\gamma_{ss}(k) = (2\pi \langle S^z\rangle^2/N^2) \sum_{q,p} V^2_{kqp}[\bar{n}_p(1 + \bar{n}_{k-q} + \bar{n}_{p+q}) - \bar{n}_{k-q}\bar{n}_{p+q}] \times \delta(E_{p+q} + E_{k-q} - E_p - E_k) \tag{17}$$

where

$$V_{kqp} = (J_q + J_{k-q-p}) - (J_{k-q} + J_{p+q})$$
$$\bar{n}_q = \langle S_q^- S_q^+\rangle/2\langle S^z\rangle = (1/4)\left[(\epsilon_{11}(q)/E_q)\coth(E_q/2k_B T) - 1\right]. \tag{18}$$

γ_{sf} is the damping which comes from the interaction between the ferromagnetically ordered and the conduction band electrons

$$\gamma_{sf}(k) = (2\pi I^2\langle S^z\rangle/N^3)\sum_{q,p,r}\{(\bar{n}_p - \bar{n}_{p+k+q})\bar{m}_{q+r+}(1-\bar{m}_{r-})$$
$$+\bar{n}_{p+k+q}(1+\bar{n}_p)(\bar{m}_{q+r+}-\bar{m}_{r-})\}\delta(E_{p+k+q}-E_p+\epsilon_{q+r+}-\epsilon_{r-}-E_k)$$
$$+(\pi I^2/4N^2)\sum_{q,p,\sigma}\{\bar{m}_{p+q\sigma}(1-\bar{m}_{p\sigma})+\bar{n}_{k-q}(\bar{m}_{p+q\sigma}-\bar{m}_{p\sigma})\}$$
$$\times\,\delta(E_{k-q}+\epsilon_{p+q\sigma}-\epsilon_{p\sigma}-E_k)$$
$$+(\pi I^2\langle S^z\rangle/2N)\sum_q(\bar{m}_{q-k+}-\bar{m}_{q-})\delta(\epsilon_{q-k+}-\epsilon_{q-}-E_k) \qquad (19)$$

where

$$\bar{m}_{q\sigma} = \langle c^+_{q\sigma}c_{q\sigma}\rangle = 1/[\exp(\epsilon_{q\sigma}/k_BT)+1].$$

γ_{sp} is the damping due to the spin–phonon interaction

$$\gamma_{sp}(k) = (2\pi\langle S^z\rangle^2/N^3)\sum_{q,p,r}\{F^2_{kqpr}[\bar{n}_r(1+\bar{n}_{q+r}+\bar{n}_p)-\bar{n}_{q+r}\bar{n}_p][(1+\bar{N}_{k-p-q})$$
$$\times\,\delta(E_{q+r}-E_r+E_p-\omega_{k-p-q}-E_k)$$
$$+\bar{N}_{k-p-q}\delta(E_{q+r}-E_r+E_p+\omega_{k-p-q}-E_k)]$$
$$+F^2_{kqpr}\bar{n}_r(1+\bar{n}_p)(1+\bar{n}_{q+r})[\delta(E_{q+r}-E_r+E_p-\omega_{k-p-q}-E_k)$$
$$-\delta(E_{q+r}-E_r+E_p+\omega_{k-p-q}-E_k)]\}$$
$$+(\pi/4N)\sum_q F^2_{kq}[(1+\bar{N}_{q-k}+\bar{n}_q)\delta(E_q-\omega_{q-k}-E_k)$$
$$+(\bar{N}_{q-k}-\bar{n}_q)\delta(E_q+\omega_{q-k}-E_k)] \qquad (20)$$

where

$$F_{kqpr} = (F(q,k-p)+F(p-r,k-q-r))-(F(k-q,p)+F(k-p+r,q+r))$$
$$\bar{N}_q = \langle a^+_q a_q\rangle = 1/[\exp(\omega_q/k_BT)-1]. \qquad (21)$$

At $T=0$

$$\gamma_k(T=0) = (\pi/4N)\sum_q F^2_{kq}\delta(E_q-\omega_{q-k}-E_k). \qquad (22)$$

is valid. We can see that at $T=0$ the spin waves are damped due to the spin–phonon interaction provided the δ-function can be satisfied. Then γ_k increases with increasing temperature T.

At temperatures close, but less than T_c the parts from the s–f and spin–phonon interaction predominate over this due to the spin–spin interaction. Therefore we have

$$\gamma_{ss}(k) \ll \gamma_{sp}(k) \ll \gamma_{sf}(k) \qquad \text{for } T \simeq T_c. \qquad (23)$$

The expressions of the damping for $T \geqslant T_c$ are:

$$\gamma_{ss}(k) = 0 \tag{24}$$

$$\gamma_{sf}(k) = (\pi I^2/4N^2) \sum_{q,p,\sigma} \bar{m}_{p+q\sigma}(1-\bar{m}_{p\sigma})\delta(E_{k-q}+\epsilon_{p+q\sigma}-\epsilon_{p\sigma}-E_k) \tag{25}$$

$$\gamma_{sp}(k) = (\pi/4N) \sum_q F_{kq}^2[(1+\bar{N}_{q-k}+\bar{n}_q)\delta(E_q-\omega_{q-k}-E_k) + (\bar{N}_{q-k}-\bar{n}_q)\delta(E_q+\omega_{q-k}-E_k)] \tag{26}$$

with $\bar{n}_q$ from (18) and E_q from (10) with $\langle S^z\rangle \to 0$.

For $k = 0$ is the term γ_{ss} which comes from the spin–spin interaction zero; only the terms due to the s–f and spin–phonon interactions give contribution to the transverse damping.

For small k-values we have

$$\gamma_{ss} \ll \gamma_{sp} \ll \gamma_{sf} \qquad k \simeq 0. \tag{27}$$

So the spin–phonon interaction gives the main contribution to the damping at temperatures $T \gtrsim T_c$ and for small wave vector k and must be taken into account if we want to obtain correct results.

The numerical calculations of the damping are in preparation and will be published elsewhere.

5. Another interaction mechanism

The calculations presented here could be extended to include other mechanisms giving interactions between spin waves and phonons. For example, it would be useful to consider the effects of spin–phonon coupling due to modulation of the crystalline field (the single-ion magnetostriction mechanism). This can give an interaction Hamiltonian which is linear in the phonon operators and quadratic in the spin operators:

$$H_{mp} = -\sum_q F_q(a_q + a_{-q}^+)S_{-q}^z. \tag{28}$$

We define the same Green function (7) as in section 2, and use the same method applied there.

The transverse spin wave energy E_k is obtained as in (10) but with the following ϵ_{11}:

$$\epsilon_{11} = g\mu_B H + (1/2\langle S^z\rangle N)\sum_q (J_q - J_{k-q})(2\langle S_q^z S_{-q}^z\rangle + \langle S_q^- S_q^+\rangle) + \Delta J_{sp}\langle S^z\rangle + (I/N^2)\sum_{q,p}(\langle S_{p-q}^- c_{p+}^+ c_{q-}\rangle + \langle S_{q-p}^z c_{p+}^+ c_{q+}\rangle - \langle S_{q-p}^z c_{p-}^+ c_{q-}\rangle). \tag{29}$$

In the RPA ϵ_{11} is then equal to

$$\epsilon_{11} = g\mu_B H + \langle S^z\rangle(J_{eff} - J_k) + I\rho. \tag{30}$$

The spin–phonon interaction causes a renormalization of the spin–spin interaction constant $J_0 \to J_{\text{eff}}$

$$J_{\text{eff}} = J_0 + \Delta J_{\text{sp}}$$
$$\Delta J_{\text{sp}} = \lim_{q\to 0}(2F_q^2/\omega_q). \tag{31}$$

Calculations yield the following expression for γ_k:

$$\gamma_k = (\pi/N)\sum_q F_{qk}^2(1+\bar{N}_q+\bar{n}_{k-q})\delta(E_{k-q}-\omega_q-E_k) + (\pi/N)\sum_q F_{qk}^2(\bar{N}_q-\bar{n}_{k-q})\delta(E_{k-q}+\omega_q-E_k)+\gamma_{\text{ss}}+\gamma_{\text{sf}} \tag{32}$$

with γ_{ss} and γ_{sf} from (17) and (19), respectively.

At $T = 0$ this simplifies to

$$\gamma_k(T=0) = (\pi/N)\sum_q F_{qk}^2\delta(E_{k-q}-\omega_q-E_k). \tag{33}$$

Hence the spin waves may be damped at zero temperature, provided the delta function in (33) can be satisfied.

At $T = T_c$ the transverse damping γ_{ss} which arises from the spin–spin interaction vanishes, whereas the damping due to the s–f and spin–phonon interaction remains finite. For $T \geqslant T_c$ we have

$$\gamma_k(T\geqslant T_c) = (\pi/N)\sum_q F_{qk}^2[(1+\bar{N}_q+\bar{n}_{k-q})\delta(E_{k-q}-\omega_q-E_k) + (\bar{N}_q-\bar{n}_{k-q})\delta(E_{k-q}+\omega_q-E_k)]+\gamma_{\text{sf}} \tag{34}$$

with γ_{sf} from (25), $\bar{n}_q$ and $\bar{N}_q$ from (18) with E_q from (29) ($\langle S^z\rangle \to 0$) and from (21), respectively.

In the case of the first spin–phonon mechanism (6) the damping for $T \geqslant T_c$ is due to the s–f and spin–phonon interactions, too.

6. Conclusions

In the present paper a self-consistent theory of the spin–phonon interaction in a ferromagnetic semiconductor was developed. The renormalized spin wave energy is obtained. The spin–phonon interaction causes a renormalization of the spin–spin interaction constant $J_0 \to J_{\text{eff}}$, which is now temperature dependent. Above T_c we have $\Delta J_{\text{sp}} = 0$. The damping is very small at low temperatures, but finite for $T = 0$. Then it increases with increasing of T. At $T = T_c$ and above T_c only the damping due to the s–f and spin–phonon interactions remains finite. For $k = 0$, $\gamma_{\text{sp}} = \gamma_{\text{ss}} = 0$, whereas γ_{sf} contributes to the transverse damping.

In the last section we consider an interaction Hamiltonian which is linear in the phonon operators and quadratic in the spin operators. The spin–phonon interaction causes a renormalization of the spin–spin interaction constant J_0. At zero temperature the spin waves are renormalized and may be damped provided the delta function can be satisfied. For $T \geqslant T_c$ only γ_{sf} and γ_{sp} remain finite. Results of the numerical calculations of the obtained expressions will be published elsewhere. It may be concluded that the spin–phonon interaction plays an important role and must be considered in order to obtain correct results in ferromagnetic semiconductors.

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